Open Bisimulation, Revisited

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Outline

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Syntax

- Processes

$$P,Q \quad ::= \quad \mathbf{0} \quad \mid \quad !\,P \quad \mid \quad P \| Q \quad \mid \quad P + Q$$
$$\mid \quad \pi.P \quad \mid \quad (\nu z)\,P \quad \mid \quad [x = y]P$$

- Prefixes

$$\pi \quad ::= \quad \tau \quad \mid \quad x(z) \quad \mid \quad \overline{x}\langle z \rangle$$

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- Only names

Late Labelled Semantics

$$\textsc{Input}\ \frac{}{a(x).P \xrightarrow{a(x)} P} \qquad \textsc{Open}\ \frac{P \xrightarrow{\bar{a}\,z} P'}{(\nu z)\,P \xrightarrow{(\nu z)\,\bar{a}\,z} P'}\ z \neq a$$

$$\textsc{Close-L}\ \frac{P \xrightarrow{a(x)} P' \qquad Q \xrightarrow{(\nu z)\,\bar{a}\,z} Q'}{P \| Q \xrightarrow{\tau} (\nu z)\,(P'\{^{z}/_{x}\} \| Q')}\ z \notin \mathrm{fn}(P)$$

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$$P$$
$$|$$
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Some questions when designing a bisimulation:

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- By what?

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$$\begin{array}{cccccc} & P & & P\sigma & \xrightarrow{\alpha} & P' \\ \sigma \triangleright D & | & & & & | \\ & Q & & Q\sigma & \xrightarrow{\alpha} & Q' \end{array}$$

Indexed by a *distinction* D.

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  - $x \mapsto u$ respects D and the updated distinction is $\{(u, y), (u, z), (y, u), (z, u)\}$
  - On the contrary, $x \mapsto u, y \mapsto u$ does not respect D

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  - otherwise, if $\alpha = (\nu z)\,\overline{a}\,z$, then $(D', P', Q') \in \mathcal{R}$ where $D' = D\sigma \cup \{z\} \otimes (\mathrm{fn}((P+Q)\sigma))$
- Distinctions are used to forbid the fusing of fresh names with other names

The lazy flavour of open

$$P \stackrel{\text{def}}{=} c(x).(\tau + \tau.\tau + \tau.[x = a]\tau)$$
$$Q \stackrel{\text{def}}{=} c(x).(\tau + \tau.\tau)$$

The lazy flavour of open

$$
\begin{aligned}
P &\stackrel{\text{def}}{=} c(x).(\tau + \tau.\tau + \tau.[x = a]\tau) \\
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- P and Q are late bisimilar but not open

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- P and Q are late bisimilar but not open
- In open, the instantiation of x can be delayed until x is used

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For these reasons, we wanted to extend open to the spi-calculus.

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- Expressions

$$\begin{array}{rcllll} E, F & ::= & x & & & \\ & \mid & (E \,.\, F) & \mid \ \pi_1(E) & \mid \ \pi_2(E) \\ & \mid & \mathsf{E}_F(E) & \mid \ \mathsf{D}_F(E) & \end{array}$$

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- Guards

$$\phi \quad ::= \quad [E = F] \quad \mid \quad [E : \mathcal{N}]$$

Evaluation of expressions and formulae

$$
\begin{aligned}
[\![a]\!] &\stackrel{\text{def}}{=} a \\
[\![\mathsf{E}_F(E)]\!] &\stackrel{\text{def}}{=} \mathsf{E}_N(M) && \text{if } [\![E]\!] = M \in \mathcal{M} \text{ and } [\![F]\!] = N \in \mathcal{M} \\
[\![\mathsf{D}_F(E)]\!] &\stackrel{\text{def}}{=} M && \text{if } [\![E]\!] = \mathsf{E}_N(M) \in \mathcal{M} \text{ and } [\![F]\!] = N \in \mathcal{M} \\
[\![E]\!] &\stackrel{\text{def}}{=} \bot && \text{in all other cases}
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[\![\mathsf{E}_F(E)]\!] & \stackrel{\text{def}}{=} & \mathsf{E}_N(M) & \text{if } [\![E]\!] = M \in \mathcal{M} \text{ and } [\![F]\!] = N \in \mathcal{M} \\
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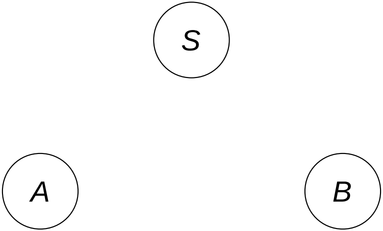
[\![E]\!] & \stackrel{\text{def}}{=} & \perp & \text{in all other cases}
\end{array}
$$

$$
\begin{array}{rcll}
[\![tt]\!] & \stackrel{\text{def}}{=} & \textbf{true} & \\
[\![\phi \wedge \psi]\!] & \stackrel{\text{def}}{=} & [\![\phi]\!] \text{ and } [\![\psi]\!] & \\
[\![[E = F]]\!] & \stackrel{\text{def}}{=} & \textbf{true} & \text{if } [\![E]\!] = [\![F]\!] = M \in \mathcal{M} \\
[\![[E : \mathcal{N}]]\!] & \stackrel{\text{def}}{=} & \textbf{true} & \text{if } [\![E]\!] = a \in \mathcal{N} \\
[\![\phi]\!] & \stackrel{\text{def}}{=} & \textbf{false} & \text{in all other cases}
\end{array}
$$

The wide-mouthed frog protocol



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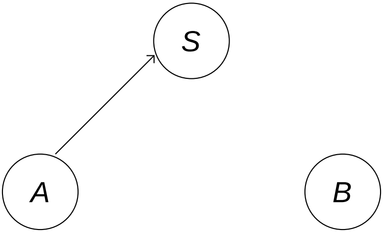

1. $A \rightarrow S : (A \,.\, \mathrm{E}_{k_{AS}}((B \,.\, k_{AB})))$

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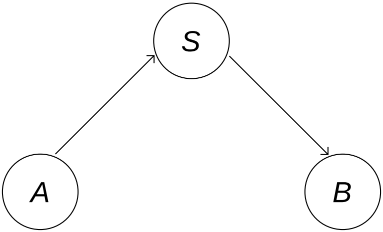

1. $A \rightarrow S : (A \,.\, \mathrm{E}_{k_{AS}}((B \,.\, k_{AB})))$
2. $S \rightarrow B : \mathrm{E}_{k_{BS}}(((A \,.\, B) \,.\, k_{AB}))$

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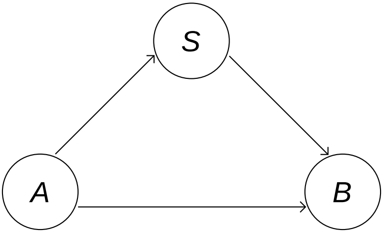

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2. $S \rightarrow B : \mathrm{E}_{k_{BS}}(((A \,.\, B) \,.\, k_{AB}))$
3. $A \rightarrow B : \mathrm{E}_{k_{AB}}(m)$

.. in spi-calculus

$$
\begin{array}{l}
(\nu k_{AS}, k_{BS}) \\
\quad (\nu k_{AB})\,\overline{S}\langle(A\,.\,\mathrm{E}_{k_{AS}}((B\,.\,k_{AB})))\rangle.\overline{B}\langle\mathrm{E}_{k_{AB}}(m)\rangle.\,\mathbf{0} \\
\quad \| B(x_1).\phi_1 B(x_2).\phi_2\,\mathbf{0} \\
\quad \| S(x_0).\phi_0 \overline{B}\langle\mathrm{E}_{k_{BS}}(((A\,.\,B)\,.\,\pi_2(\mathrm{D}_{k_{AS}}(\pi_2(x_0)))))\rangle.\,\mathbf{0}
\end{array}
$$

.. in spi-calculus

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\begin{array}{l}
(\nu k_{AS}, k_{BS}) \\
\quad (\nu k_{AB})\, \overline{S}\langle (A\,.\,\mathrm{E}_{k_{AS}}((B\,.\,k_{AB})))\rangle.\overline{B}\langle \mathrm{E}_{k_{AB}}(m)\rangle.\,\mathbf{0} \\
\quad \| B(x_1).\phi_1 B(x_2).\phi_2\, \mathbf{0} \\
\quad \| S(x_0).\phi_0 \overline{B}\langle \mathrm{E}_{k_{BS}}(((A\,.\,B)\,.\,\pi_2(\mathrm{D}_{k_{AS}}(\pi_2(x_0))))))\rangle.\,\mathbf{0}
\end{array}
$$

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\begin{array}{lcl}
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- Abadi and Gordon have introduced environment-sensitive bisimulation.

Outline

Different kinds of free names

$$P \stackrel{\text{def}}{=} a(x).(\nu k)\,\overline{b}\langle k\rangle.\overline{x}\langle k\rangle.\mathbf{0}$$

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C	V	$\prec$
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- The corresponding distinction is

$$D(C, V, \prec) \stackrel{\text{def}}{=} C^{\neq} \cup \{n \neq x \mid n \in C \wedge \neg(n \prec x)\}$$

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- if e is an environment, then open $D(e)$-bisimilarity is an e-congruence

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- For example, if $h = \{(a,a),(k,k)\}$, we have $(\mathrm{E}_k(a), \mathrm{E}_k(a)) \in \mathcal{S}(h)$
- In general, $\mathcal{S}(h)$ is not a hedge since it is not finite.

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- The irreducibles $\mathcal{I}(h)$ is a “minimal” hedge “equivalent” to $\mathcal{A}(h)$

Late hedged bisimulation

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- A “symmetric” relation $\mathcal{R} \subset \mathcal{H} \times \mathcal{P} \times \mathcal{P}$ is a late hedged bisimulation if for all $(h, P, Q) \in \mathcal{R}$,
  - h is consistent
  - if $P \xrightarrow{\alpha} P'$ and $\mathrm{ch}(\alpha) \in \pi_1(h)$ then $Q \xrightarrow{\beta} Q'$ and
    1. if $\alpha = \tau$, then $\beta = \tau$ and $(h, P', Q') \in \mathcal{R}$
    2. if $\alpha = a(x)$, then $\beta = b(x)$ and
       $(a, b) \in \mathcal{S}(h)$
       for all $B \subset \mathcal{N} \times \mathcal{N}$ consistent (and minimal, and fresh)
       for all $(M, N) \in \mathcal{S}(h \cup B)$, $(h \cup B, P'\{^M/_x\}, Q'\{^N/_x\}) \in \mathcal{R}$
    3. if $\alpha = (\nu\tilde{c})\,\overline{a}\,M$, then $\beta = (\nu\tilde{d})\,\overline{b}\,N$ and
       $(a, b) \in \mathcal{S}(h)$
       $(\mathcal{I}(h \cup \{(M, N)\}), P', Q') \in \mathcal{R}$

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$$P \stackrel{\text{def}}{=} (\nu k)\,(\nu m)\,\overline{a}\langle \mathsf{E}_k(m)\rangle.a(x).(\overline{a}\langle k\rangle \parallel [x = k]\overline{a}\langle a\rangle)$$

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- The definition obtained is sound.

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Future work

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  - Congruence properties?

Thank you!

Thank you!

Questions?

Bibliography

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e-respectful contexts

If $e = (O, V, \prec)$, a context $C[\cdot]$ respects e if it is generated by

$$
\begin{array}{rcll}
C_N[\cdot] & ::= & [\cdot] & \text{if } N = \emptyset \\
& | & P \| C_N[\cdot] \,|\, C_N[\cdot] \| P & \\
& | & P + C_N[\cdot] \,|\, C_N[\cdot] + P & \\
& | & !\, C_N[\cdot] & \\
& | & \phi C_N[\cdot] & \\
& | & (\nu x)\, C_{N \setminus \{x\}}[\cdot] & \\
& | & \overline{a}\langle z \rangle . C_N[\cdot] & \\
& | & a(x).C_N[\cdot] & \text{if } x \notin O \cup V \\
& | & a(x).C_{N \cup N'}[\cdot] & \text{if } x \in V \text{ and } N' = \{ n \in O \mid \neg n \prec x \}
\end{array}
$$

with $N \subset O$ and $C_\emptyset[\cdot]$ as start symbol.